Non-factorized genuine twist-3 in exclusive electro-production of vector mesons

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We present an analysis of genuine twist-3 quark contributions to the amplitude of exclusive electro-production of transversely polarized vector mesons. Using the formalism based on the momentum representation we calculated all the genuine twist-3 terms of quark contributions to $\gamma^*_T \to \rho_T$ amplitude. We found that these terms can not be factorized owing to the existence of the infrared divergencies in the amplitude of hard sub-processes.

The electro-production of transverse polarized vector mesons hadron$(p_1)+\gamma^*(q) \to \rho(p)+$hadron$(p_2)$ provides the well-known example of QCD factorization breaking. Indeed, the factorization is valid in the case of longitudinally polarized ρ-meson production unless the high twist effects are included [1]. However, the description of transversely polarized meson production is strongly complicated due to the existence of infrared divergencies in amplitudes, breaking down the factorization (see e.g. [3], [4] and references therein).

The amplitude of transverse vector meson production corresponds to the $1/Q$ suppressed contributions in comparison with the longitudinal vector meson case [1]. At the same time, the resent experiments show that the transverse mesons production amplitudes do sensible contributions even at moderate virtualities Q^2 [5]. So, to describe these processes we are forced to deal with the taking into account the terms of $1/Q$ − power while, on the other hand, it should yield to the problems related to the factorization theorem. In Ref. [2], the analysis of twist-2 amplitudes for hard exclusive lepto-production of mesons in terms of generalized parton distributions (GPDs or SPDs) was presented. Recently authors of [3] discussed the factorization problems in the electro-production of light vector mesons from transversely polarized photons. They have taken into account just kinematical twist-3 terms. Also the helicity flip amplitude of transversely polarized vector mesons production was considered within the kinematical Wandzura–Wilczek approximation [6].

Although the kinematical and dynamical (genuine) higher twists contributions are, generally speaking, independent, there are notable exceptions in some kinematical regions. In Deep Inelastic scattering at $x_B \to 1$ the kinematical higher twist terms, described by Nachtmann variable, lead to inconsistencies unless the genuine higher twists are taken into account [7]. One cannot exclude, that the genuine higher twists may cure, at least partially, the problem arising from the treatment of the end-point regions in hard electro-production.

Thus we want to study the role of genuine twist-3 contributions in the factorization theorem breaking. We adhere the approach based on the momentum representation the basic stages of which are expounded in previous papers [8]- [10].

We now derive the electro-production amplitude of transversely polarized ρ-meson in terms of the coefficient function and "soft" functions. First let us introduce the kinematics in such a manner: the p is momentum of transversely polarized ρ-meson and its polarization vector is e^T; the momentum of virtual photon is denoted by

$q(Q^2 = -q^2)$. We assume that the initial hadron momentum p_1 and final hadron momentum p_2 are collinear, and $p_1^2 = p_2^2 = t = 0$, neglecting all the relevant higher twists contributions. In addition we neglect squares of meson masses, restricting ourselves to twist 3 contributions. With the help of p_1 and p_2 momenta we build the general relative momentum $\overline{P} = (p_2 + p_1)/2$ and transfer momentum $\Delta = p_2 - p_1$. In our approximation $\overline{P}^2 = \Delta^2 = 0$.

We introduce the parameterizations of all the matrix elements needed for calculations of amplitudes. We use the axial gauge $n \cdot A = 0$, where $n^2 = 0, n \cdot p = 1$ In terms of the light-cone basis vectors the ρ-meson–to–vacuum matrix elements can be written as (keeping just the terms up to the twist-3 order):

$$\langle p|\bar{\psi}(0)\gamma_\mu\psi(z)|0\rangle \stackrel{\mathcal{F}}{=} \varphi_1(y)p_\mu + \varphi_3(y)e^T_\mu, \langle p|\bar{\psi}(0)\gamma_\mu \overleftrightarrow{\partial^T_\rho} \psi(z)|0\rangle \stackrel{\mathcal{F}}{=} \varphi^T_1(y)p_\mu e^T_\rho, \tag{1}$$

$$\langle p|\bar{\psi}(0)\gamma_5\gamma_\mu\psi(z)|0\rangle \stackrel{\mathcal{F}}{=} i\varphi_A(y)\varepsilon_{\rho\alpha\beta\delta}e^T_\alpha p_\beta n_\delta,$$
$$\langle p|\bar{\psi}(0)\gamma_5\gamma_\mu \overleftrightarrow{\partial^T_\rho} \psi(z)|0\rangle \stackrel{\mathcal{F}}{=} i\varphi^T_A(y)p_\mu\varepsilon_{\rho\alpha\beta\delta}e^T_\alpha p_\beta n_\delta, \tag{2}$$

$$\langle p|\bar{\psi}(0)\gamma_\mu g A^T_\rho(z_2)\psi(z_1)|0\rangle \stackrel{\mathcal{F}}{=} \Phi(y_1, y_2)p_\mu e^T_\rho,$$
$$\langle p|\bar{\psi}(0)\gamma_5\gamma_\mu g A^T_\rho(z_2)\psi(z_1)|0\rangle \stackrel{\mathcal{F}}{=} iJ(y_1, y_2)p_\mu\varepsilon_{\rho\alpha\beta\delta}e^T_\alpha p_\beta n_\delta, \tag{3}$$

where $\stackrel{\mathcal{F}}{=}$ denotes the Fourier transformation with measure $(z_i = \lambda_i n)$

$$\begin{aligned} dy\, e^{-iy\,pz} \quad & \text{for quark correlators,} \\ dy_1\, dy_2\, e^{-iy_1\,pz_1 - i(1-y_2-y_1)\,pz_2} \quad & \text{for quark} - \text{gluon correlators.} \end{aligned} \tag{4}$$

In the adopted collinear limit we directly obtain that $p_2 = (1-\xi)/(1+\xi)p_1 = \kappa p_1$. Therefore, $u(p_2) = \sqrt{\kappa}u(p_1)$. This lead to the symmetrical nucleon spinor forms like $\bar{u}(p_1)\sqrt{\kappa}\hat{n}u(p_1)$ whose calculation is straightforward. As a result we are able to introduce the parameterization of relevant matrix elements for nucleons. Further, keeping the twist-2 terms only, we write

$$\langle p_2|\bar{\psi}(0)\gamma_\mu\psi(\tilde{z})|p_1\rangle \stackrel{\mathcal{F}}{=} H_1(x)\overline{P}_\mu \tag{5}$$

for the nucleon–nucleon matrix element of pure quark correlator .

In this paragraph we compute the quark contributions to the production amplitude which are generated by the diagrams with the quark legs coming from the nucleon blob. The calculating of the simplest Feynman diagrams give the following expression:

$$\begin{aligned} \mathcal{A}^{(q),\,\gamma^*_T\to\rho_T}_{1,\,\mu} = 4\,\frac{C_F}{N_c}\,\frac{e^T_\mu}{Q^2}\int_{-1}^{1} dx H_1(x)\left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right] \\ \int_0^1 \frac{dy}{y(1-y)}\Big(\varphi_A(y) + \varphi_3(y)\Big). \end{aligned} \tag{6}$$

Next, summing the diagrams that do contribute to the genuine twist-3, we finally obtain the expression for the quark distribution amplitude. We write

$$\begin{aligned} \mathcal{A}^{(q),\,\gamma^*_T\to\rho_T}_{2,\,\mu} = 4\,\frac{C_F}{N_c^2-1}\,\frac{e^T_\mu}{Q^2}\Bigg\{\xi\int_{-1}^{1} dx H_1(x)\left[\frac{1}{(x+\xi-i\epsilon)^2} + \frac{1}{(x-\xi+i\epsilon)^2}\right]\mathcal{I}^{(q)}_1 + \\ \int_{-1}^{1} dx H_1(x)\left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right]\mathcal{I}^{(q)}_2\Bigg\}, \end{aligned} \tag{7}$$

where

$$\mathcal{I}_1^{(q)} = \int_0^1 dy_1\, dy_2 \Bigg\{ \tilde{J}(y_1, y_2) \Bigg[4C_F \left(\frac{1}{(1-y_1)^2} - \frac{1}{(1-y_2)^2} \right) +$$
$$C_A \left(\frac{1}{y_2(1-y_1)} - \frac{1}{y_1(1-y_2)} \right) + \frac{2C_F - C_A}{y_1+y_2} \left(\frac{1}{y_1} - \frac{1}{y_2} \right) \Bigg] -$$
$$\tilde{\Phi}(y_1, y_2) \Bigg[4C_F \left(\frac{1}{(1-y_1)^2} + \frac{1}{(1-y_2)^2} \right) + C_A \left(\frac{1}{y_2(1-y_1)} + \frac{1}{y_1(1-y_2)} \right) +$$
$$\frac{2C_F - C_A}{y_1+y_2} \left(\frac{1}{y_1} + \frac{1}{y_2} \right) \Bigg] \Bigg\}; \tag{8}$$

$$\mathcal{I}_2^{(q)} = \int_0^1 dy_1\, dy_2 \Bigg\{ \tilde{J}(y_1, y_2) \Bigg[C_F \left(\frac{1}{y_2(1-y_1)} - \frac{1}{y_1(1-y_2)} \right) + \frac{2C_F - C_A}{y_1+y_2} \left(\frac{1}{y_2} - \frac{1}{y_1} \right) \Bigg] -$$
$$\tilde{\Phi}(y_1, y_2) \Bigg[C_F \left(\frac{1}{y_2(1-y_1)} + \frac{1}{y_1(1-y_2)} \right) + \frac{2C_F - C_A}{y_1+y_2} \left(\frac{1}{y_2} + \frac{1}{y_1} \right) \Bigg] \Bigg\}. \tag{9}$$

Here we introduced the notation for the general parameterizing functions $\tilde{\Phi}$ and $\tilde{J}$ consisting of the kinematical and genuine (dynamical) twist-3 parts, *i.e.*, for example,

$$\tilde{\Phi}(y_1, y_2) = \varphi_1^T(y_1)\delta(y_1 - y_2) + \Phi(y_1, y_2). \tag{10}$$

The structure integrals of the quark distribution amplitude possess poles of second order also. This fact lead to the vulnerability of factorization theorem unless the functions $\tilde{\Phi}$ and $\tilde{J}$ vanish at $y_i \to 1$ or $y_i \to 0$ more rapidly then the first power of $(1 - y_i)$ or y_i. Within our formalism we have reproduced, also, the results for the kinematical twist 3 gluon distribution amplitude obtained in [2].

In summary, we have computed the total quark contributions to the transversely polarized ρ-meson electroproduction. We have found the novel singular terms of the genuine twist-3 to the quark contribution amplitude, which must be included in the general analysis of problems related with the factorization theorem breaking.

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[1] J.C. Collins, L. Frankfurt, M. Strikman, Phys.Rev. **D56**, 2982 (1997)

[2] L. Mankiewicz, G. Piller, T.Weigl, Acta Phys.Polon. **B29**, 1235 (1998) [hep-ph/9711227]

[3] L. Mankiewicz, G. Piller, Phys.Rev. **D61**,074013 (2000) [hep-ph/9905287]

[4] A.V. Radyushkin, Phys. Rev. **D56**, 5524 (1997) [hep-ph/9704207].

[5] ZEUS Collaborations (J. Breitweg et al.) Eur.Phys.J. **C12**, 393 (2000) [hep-ex/9908026]; HERMES Collaboration (A. Airapetian et al.) Phys.Lett. **B513**, 301 (2001) [hep-ex/0102037]

[6] N. Kivel, Phys.Rev. **D65**, 054010 (2002) [hep-ph/0107275]

[7] A. De Rujula, H. Georgi and H. D. Politzer, Phys. Rev. D **15**, 2495 (1977).

[8] I.V. Anikin, B. Pire, O.V. Teryaev, Phys.Rev. **D62**, 071501 (2000) [hep-ph/0003203]

[9] I.V. Anikin, O.V. Teryaev, Phys.Lett. **B509**, 95 (2001) [hep-ph/0102209]

[10] I.V. Anikin, D. Binosi, R. Medrano, S. Noguera and V. Vento, Eur. Phys. J. **A14**, 95 (2002) [hep-ph/0109139]

[11] P. Ball, V. Braun, Phys.Rev. **D54**, 2182 (1996); P. Ball et al, Nucl.Phys. **B529**, 323 (1998)